

# Finite frequency noise of a superconductor/ferromagnet quantum point contact

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(Dated: November 27, 2008)

We have calculated the finite-frequency current noise of a superconductor-ferromagnet quantum point contact (SF QPC). This signal is qualitatively affected by the spin-dependence of interfacial phase shifts (SDIPS) acquired by electrons upon reflection on the QPC. For a weakly transparent QPC, noise steps appear at frequencies or voltages determined directly by the SDIPS. These steps can occur at experimentally accessible temperatures and frequencies. Finite frequency noise is thus a promising tool to characterize the scattering properties of a SF QPC.

PACS numbers: 74.50.+r, 72.25.-b, 72.70.+m

Spin-dependent transport in mesoscopic structures with ferromagnetic elements is generating a growing interest, due to the great variety of behaviors allowed by the lifting of spin-degeneracy and the possibilities of applications in the field of nanospintronics<sup>1</sup>. In this context, the study of finite-frequency noise ( $\mathcal{FFN}$ ) has raised little attention so far<sup>2</sup>. However, a circuit is characterized by a larger number of parameters when spin-degeneracy is lifted, and current noise can thus be particularly useful to obtain more information on those parameters. Recently, superconductor/ferromagnet ( $SF$ ) quantum points contacts (QPCs) have raised much interest as a possible source of information on the properties of  $F$  materials. The average current flowing through  $SF$  QPCs has been intensively studied (see e.g. Ref. 3), and the zero-frequency current noise of a single-channel  $SF$  QPC has been recently calculated<sup>4</sup>. It has been shown that these quantities are affected by the spin-polarization of the QPC transmission probabilities, but also by the Spin-Dependence of Interfacial Phase Shifts (SDIPS) acquired by electrons upon reflection on the  $SF$  interface. Regarding  $\mathcal{FFN}$  in QPCs, the normal metal/normal metal ( $NN$ ) case has been studied theoretically<sup>5</sup> and experimentally<sup>6</sup>, and the  $SN$  case has been addressed theoretically<sup>7</sup>, but spin-dependent transport has not been considered so far.

In this Letter, we investigate theoretically the  $\mathcal{FFN}$  of a  $SF$  QPC. Our main result is that, for a weakly transparent contact, the  $\mathcal{FFN}$  is qualitatively affected by the SDIPS. More precisely, some characteristic noise steps appear at various frequencies or voltages determined directly by the SDIPS value. These steps occur at experimentally accessible temperatures and frequencies. Therefore,  $\mathcal{FFN}$  is a promising tool to characterize the scattering properties of a  $SF$  QPC. More generally, this work illustrates that  $\mathcal{FFN}$  can provide an interesting insight in spin-dependent transport.

We define the non-symmetrized current noise  $S(V, \omega) = \int_{-\infty}^{+\infty} \langle \Delta \hat{I}(0) \Delta \hat{I}(t) \rangle e^{i\omega t} dt$  of a  $SF$  contact, with  $\Delta \hat{I}(t) = \hat{I}(t) - \langle I \rangle$  and  $\hat{I}$  the current operator of

the contact. In general,  $S(V, \omega) \neq S(V, -\omega)$ , because  $S(V, \omega)$  describes photon emission (absorption) processes between the system and a detector for  $\omega > 0$  ( $\omega < 0$ ). The excess noise  $S_{ex}(\omega) = S(V, \omega) - S(V = 0, \omega)$  is often the relevant observable in experiments<sup>6,8,9,10,11,12</sup>. Interestingly,  $S_{ex}$  has already been measured for both signs of  $\omega$ <sup>12</sup>. We first consider a general ballistic  $SF$  contact, with  $S$  a conventional superconductor described by the Bogoliubov-De Gennes (BdG) equations<sup>13</sup>. A bias voltage  $V$  is applied to the contact, so that the Fermi level of  $S[F]$  can be set to  $\mu_S = 0$  [ $\mu_F = -eV$ ]. We denote with  $\sigma \in \{\uparrow, \downarrow\}$  an electronic spin component collinear to the polarization of  $F$ . Electron states in the  $\sigma$ -spin band [denoted  $(e, \sigma)$ ] at an energy  $\varepsilon$  and hole states in the  $-\sigma$  spin band [denoted  $(h, -\sigma)$ ] at an energy  $-\varepsilon$  are coupled coherently by Andreev reflections occurring at the  $SF$  interface. Using the scattering approach<sup>5</sup>, we find

$$S(V, \omega) = \frac{e^2}{h} \sum_{\substack{M, P, \\ \gamma, \eta, \alpha, \beta}} \lambda(\alpha) \lambda(\beta) \int_{-\infty}^{+\infty} d\varepsilon P_{M, P}^{\gamma, \eta}(\varepsilon, \varepsilon + \omega) \\ \text{Tr} \left[ \mathcal{A}_{F, M, P}^{\alpha, \gamma, \eta}(\varepsilon, \varepsilon + \hbar\omega) \mathcal{A}_{F, P, M}^{\beta, \eta, \gamma}(\varepsilon + \hbar\omega, \varepsilon) \right]. \quad (1)$$

Here, capital Latin indices correspond to the lead  $F$  or  $S$  and Greek indices correspond to the electron or hole band of the space  $\mathcal{E}_\sigma = \{(e, \sigma), (h, -\sigma)\}$ . We use  $\mathcal{A}_{F, M, P}^{\gamma, \alpha, \eta}(\varepsilon, \varepsilon') = \mathbb{I}_{F, \gamma} \delta_{F, M} \delta_{F, P} \delta_{\gamma, \alpha} \delta_{\gamma, \eta} - \left( \mathcal{S}_{F, M}^{\gamma, \alpha}(\varepsilon) \right)^\dagger \mathcal{S}_{F, P}^{\gamma, \eta}(\varepsilon')$ , where  $\mathcal{S}_{F, M}^{\gamma, \alpha}(\varepsilon)$  is the scattering matrix accounting for the transmission of quasiparticles with energy  $\varepsilon$  from a state of type  $\alpha$  in lead  $M$  to a state of type  $\gamma$  in lead  $F$ . We define  $\lambda[(e, \sigma)] = -1$ ,  $\lambda[(h, -\sigma)] = +1$  and  $P_{M, P}^{\gamma, \eta}(\varepsilon, \varepsilon') = [1 - f_M^\gamma(\varepsilon)] f_P^\eta(\varepsilon')$  with  $f_M^\gamma(\varepsilon) = (1 + \exp[(\varepsilon + \lambda[\gamma]\mu_M)/k_B T])^{-1}$ , and  $T$  the temperature. We note  $e = |e|$  the absolute value of the electron charge and  $\mathbb{I}_{F, \gamma}$  the identity matrix in the subspace of states of type  $\gamma$  of lead  $F$ . In the limit  $T = 0$ , using the symmetry of the BdG equations (see

Ref. 4, appendix A) and the unitarity of  $\mathcal{S}(\varepsilon)$ , one finds

$$S_{ex}(\omega) = S_d(\omega) + \frac{e^2}{h} \left( \int_{-\hbar|\omega|}^0 d\varepsilon \text{Tr}[\tilde{F}_1(\varepsilon, \omega, V)] + \int_{-e|V|}^{-\hbar|\omega|} d\varepsilon \text{Tr}[\tilde{F}_2(\varepsilon, \omega, V)] \right) \quad (2)$$

for  $0 < |\hbar\omega| < e|V|$ ,

$$S_{ex}(\omega) = S_d(\omega) + \frac{e^2}{h} \int_{-e|V|}^{e|V|-\hbar|\omega|} d\varepsilon \text{Tr}[\tilde{F}_1(\varepsilon, \omega, V)] \quad (3)$$

for  $eV < |\hbar\omega| < 2e|V|$ , and  $S_{ex}(\omega) = S_d(\omega)$  for  $|\hbar\omega| > 2e|V|$ , with

$$S_d(\omega) = \frac{2e^2}{h} \theta(-\omega) \int_{-e|V|}^0 d\varepsilon \text{Re} \left[ \text{Tr} \left[ \tilde{F}_3(\varepsilon, \omega, V) \right] \right] \quad (4)$$

and  $\theta(\omega) = (1 + \text{sign}(\omega))/2$ . We define, for with  $i \in \{1, 2, 3\}$ ,  $\tilde{F}_i(\varepsilon, \omega, V) = \sum_{\sigma} F_{i,\sigma}^{\text{sign}[V]\varepsilon, \text{sign}[V]|\omega|}$ , with

$$F_{1,\sigma}^{\varepsilon, \omega} = s_{ee}^{\sigma}(\varepsilon) s_{ee}^{\sigma\dagger}(\varepsilon) s_{eh}^{\sigma}(\varepsilon + \hbar\omega) s_{eh}^{\sigma\dagger}(\varepsilon + \hbar\omega) - \text{Re}[s_{ee}^{\sigma}(\varepsilon) s_{he}^{\sigma\dagger}(\varepsilon) s_{hh}^{\sigma}(\varepsilon + \hbar\omega) s_{eh}^{\sigma\dagger}(\varepsilon + \hbar\omega)] \quad (5)$$

$$F_{2,\sigma}^{\varepsilon, \omega} = 2 \text{Re}[s_{ee}^{\sigma\dagger}(\varepsilon) s_{ee}^{\sigma}(\varepsilon + \hbar\omega) s_{he}^{\sigma\dagger}(\varepsilon + \hbar\omega) s_{he}^{\sigma}(\varepsilon)] + \sum_{a \in \{e, h\}} s_{ae}^{\sigma}(\varepsilon) s_{ae}^{\sigma\dagger}(\varepsilon) (\mathbb{I}_{\sigma} - s_{ae}^{\sigma}(\varepsilon + \hbar\omega) s_{ae}^{\sigma\dagger}(\varepsilon + \hbar\omega)) \quad (6)$$

and

$$F_{3,\sigma}^{\varepsilon, \omega} = \sum_{a \in \{e, h\}} \lambda_a s_{ae}^{\sigma\dagger}(\varepsilon) [s_{ae}^{\sigma}(\varepsilon - \hbar\omega) - s_{ae}^{\sigma}(\varepsilon + \hbar\omega)]. \quad (7)$$

We have used above  $s_{ab}^{\sigma}(\varepsilon) = S_{F,F}^{p(a,\sigma), p(b,\sigma)}(\varepsilon)$ , with  $p(e[h], \sigma) = (e[h], \pm\sigma)$ ,  $\lambda_{e[h]} = \mp 1$ , and  $\mathbb{I}_{\sigma} = \mathbb{I}_{F,(e,\sigma)}$ . At last,  $S$  can be obtained from  $S_{ex}$  using

$$\frac{S(V=0, \omega)h}{2e^2} = \theta(-\omega) \sum_{\sigma} \int_0^{-\hbar\omega} d\varepsilon \text{Re} [\text{Tr}[K_{\sigma}^{\omega}(\varepsilon)]] \quad (8)$$

with  $K_{\omega}^{\sigma}(\varepsilon) = \mathbb{I}_{\sigma} - \sum_a \lambda_a s_{ae}^{\sigma\dagger}(\varepsilon) s_{ae}^{\sigma}(\varepsilon + \hbar\omega)$ .

In the following, we consider a single channel  $SF$  QPC, which can be modeled as a clean  $SN$  contact in series with a  $NF$  contact, with the length of  $N$  tending to zero<sup>14</sup>. The scattering matrix  $\mathcal{P}^{\sigma}$  of the  $NF$  contact can be expressed as  $\mathcal{P}_{FF(NN)}^{\sigma} = r_{\sigma} \exp[i\varphi_{FF(NN)}^{\sigma}]$  and  $\mathcal{P}_{NF(FN)}^{\sigma} = t_{\sigma} \exp[i\varphi_{NF(FN)}^{\sigma}]$ , with  $r_{\sigma}^2 + t_{\sigma}^2 = 1$  and  $\varphi_{NN}^{\sigma} + \varphi_{FF}^{\sigma} = \varphi_{NF}^{\sigma} + \varphi_{FN}^{\sigma} + \pi [2\pi]$ . In general, the scattering phases  $\varphi_{ij}^{\sigma}$ , with  $(i, j) \in \{N, F\}^2$ , are spin-dependent because the QPC displays a spin-dependent scattering potential. This so-called Spin-Dependence of Interfacial Phase Shifts (SDIPS) modifies qualitatively the behavior of the QPC. A step approximation for the gap  $\Delta$  of  $S$  leads to an  $SN$  Andreev reflection amplitude  $\gamma = (\varepsilon - i\sqrt{\Delta^2 - \varepsilon^2})/\Delta$  for  $|\varepsilon| < \Delta$  and  $\gamma =$

$\circ$	$-eV - \tilde{\Delta}$	$+$	$-\tilde{\Delta}$	$\square$	$\tilde{\Delta} - eV$	$\star$	$eV - \tilde{\Delta}$	$\blacktriangleleft$	$\tilde{\Delta} + \tilde{\Delta}$
$\triangleleft$	$-\tilde{\Delta} - \tilde{\Delta}$	$\boxplus$	$-\tilde{\Delta}$	$\times$	0	$\blacksquare$	$eV - \tilde{\Delta}$	$\bullet$	$eV + \tilde{\Delta}$

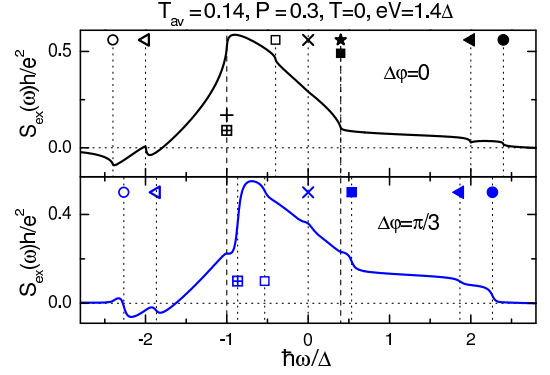


FIG. 1: Excess noise  $S_{ex}$  versus  $\omega$ , for  $V = 1.4\Delta$ ,  $T = 0$ , and different values of the SDIPS parameter  $\Delta\varphi$ . The symbols correspond to frequencies defined in the above table.

$(\varepsilon - \text{sgn}(\varepsilon)\sqrt{\varepsilon^2 - \Delta^2})/\Delta$  for  $|\varepsilon| > \Delta$ <sup>15</sup>. The resulting scattering matrix  $\mathcal{S}(\varepsilon)$  can be found in Ref. 4, Appendix B. Importantly, the elements of  $\mathcal{S}(\varepsilon)$  involve the denominator  $D_{\sigma}(\varepsilon) = 1 - \gamma^2 r_{\sigma} r_{-\sigma} e^{i(\varphi_{NN}^{\sigma} - \varphi_{NN}^{-\sigma})}$ , which accounts for iterative reflection processes between the  $SN$  and  $NF$  interfaces (Andreev bound states). A quasiparticle can interfere with itself after two back and forth travels between  $S$  and  $F$ , one as an electron ( $e, \sigma$ ) and one as a hole ( $h, -\sigma$ ), which leads to a dependence of  $D_{\sigma}$  on the SDIPS parameter  $\Delta\varphi = \varphi_{NN}^{\sigma} - \varphi_{NN}^{-\sigma}$ . We have checked analytically that  $S(V, \omega)$  depends on the phases of  $\mathcal{P}^{\sigma}$  through  $\Delta\varphi$  only, for all values of  $\omega$ ,  $V$  and  $T$ . For simplicity, we will consider below that  $t_{\sigma}$  and  $\Delta\varphi$  are independent of  $\varepsilon$  and  $V$ , so that the energy dependence of  $\mathcal{S}(\varepsilon)$  stems only from  $\gamma$ . We will mainly focus on the regime of weak transparency  $T_{av} = (t_{\uparrow}^2 + t_{\downarrow}^2)/2 \ll 1$ , with a finite polarization  $P = (t_{\uparrow}^2 - t_{\downarrow}^2)/(t_{\uparrow}^2 + t_{\downarrow}^2)$ . We will use  $T = 0$  except in Fig. 3, right panel.

We now study the frequency-parity of  $S_{ex}$ . For  $0 < e|V|, |\hbar\omega| \ll \Delta$ , the function  $S_d(\omega)$  can be neglected from Eqs. (2-3), so that one finds  $S_{ex}(\omega) = s_0(|V| - \hbar|\omega|/2e)\theta(2e|V| - \hbar|\omega|)$ , with  $s_0 = 4(e^3/h) \sum_{\sigma} \text{Tr}[s_{he}^{\sigma\dagger}(0) s_{he}^{\sigma}(0) (1 - s_{he}^{\sigma\dagger}(0) s_{he}^{\sigma}(0))]$  the zero-frequency differential noise. In this regime,  $S_{ex}$  is thus an even function of  $\omega$ , determined by zero-frequency properties only. The behavior of  $S_{ex}$  is much richer for  $eV > \Delta$ . Fig. 1 shows  $S_{ex}$  versus  $\omega$ , for  $eV = 1.4\Delta$  and different values of  $\Delta\varphi$ . We find that  $S_{ex}$  is not an even function of  $\omega$  anymore, because Andreev reflections make  $\mathcal{S}(\varepsilon)$  strongly energy-dependent [see Eq. (7)]<sup>16</sup>. Hence, it can be useful to measure  $S_{ex}$  for both signs of  $\omega$ , to gain additional information on the QPC. Interestingly,  $S_{ex}(\omega) \neq S_{ex}(-\omega)$  has already been observed in a Josephson junction<sup>12</sup>. We now focus on the sign of  $S_{ex}$ . For  $\omega > 0$  and  $T = 0$ , one always finds  $S_{ex} \geq 0$ , because

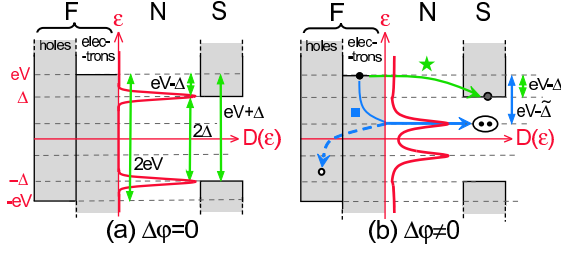


FIG. 2: Energy diagram of the *SF* QPC, for  $\Delta\varphi \neq 0$ ,  $eV > \Delta$ ,  $\Delta\varphi = 0$  (panel a) and  $\Delta\varphi \neq 0$  (panel b). The various columns represent the (hole) electron band in *F*, filled (down) up to the energy  $(-)eV$ , and the density of states in *S*, which presents a gap for  $\varepsilon \in [-\Delta, \Delta]$ . The resonance function  $D(\varepsilon) = |D_{\uparrow}(\varepsilon)|^{-2} + |D_{\downarrow}(\varepsilon)|^{-2}$  is shown with red full lines. In panel (a), the different frequencies  $\omega$  at which  $S_{ex}$  is singular are represented with green arrows. In panel (b), we show with blue and green arrows transfer processes contributing to the ★ and ■ features of Fig. 1.

emission processes are forbidden for  $V = 0$  [see Eq. (8)], so that  $S_{ex} = S$  at positive frequencies. However, nothing forbids to have  $S_{ex} < 0$  for  $\omega < 0$ , as illustrated by Fig. 1. Note, that  $S_{ex} < 0$  has already been predicted for a non-interacting conductor with energy-dependent transmission probabilities<sup>17</sup> and an interacting quantum wire<sup>18</sup>. However, in experimental works such a behavior has not been reported yet.

We now study the frequency dependence of  $S_{ex}$ . For  $\Delta\varphi = 0$  (top panel of Fig. 1),  $S_{ex}$  shows various singularities, which can be understood from Eq. (1). The current noise at frequency  $\omega$  involves a coupling between quasiparticles states with energy  $\varepsilon_1 = \varepsilon$  and  $\varepsilon_2 = \varepsilon + \hbar\omega$ . The occupation of states in *F* is discontinuous at  $\varepsilon = \pm eV$ , and the density of states in *S* is singular at  $\varepsilon = \pm\Delta$ . This allows singularities of  $S_{ex}$  at frequencies  $\omega = (\varepsilon_1 - \varepsilon_2)/\hbar$  with  $(\varepsilon_1, \varepsilon_2) \in \{eV, -eV, \Delta, -\Delta\}^2$  i.e.  $\hbar|\omega| \in \{0, |\Delta \pm eV|, 2\Delta, 2e|V|\}$ , as already found by Ref. 7 for a *S/N* QPC with  $\omega > 0$ <sup>19</sup>. These different frequencies can be represented on the energy diagram of the circuit (see Fig. 2,a for the case  $eV > \Delta$ ). In Fig. 1, we find that  $S_{ex}$  displays an extra noise singularity at  $\hbar\omega = -\Delta$ , because it involves  $S(V = 0, \omega)$ . For  $\Delta\varphi \neq 0$ , the denominator  $D_{\sigma}(\varepsilon)$  is resonant at subgap energies  $\varepsilon = \sigma\tilde{\Delta}$  with  $\tilde{\Delta} = \Delta \cos(\Delta\varphi/2)$ . This reveals that subgap resonances occur in the *N* layer<sup>4</sup>. This effect modifies qualitatively the behavior of the QPC: in Fig. 1, bottom panel, noise steps appear at  $\hbar|\omega| \in \{|\tilde{\Delta} \pm eV|, \Delta + \tilde{\Delta}\}$  and  $\hbar\omega = -\tilde{\Delta}$ , marked with the symbols ○, ◁, ▢, □, ■, ◀, and ●. These steps are due to particle transfer processes which involve the subgap resonance in the *N* layer. For instance, the step occurring at  $\omega = eV - \tilde{\Delta}$ , marked with ■ in Fig. 1, corresponds e.g. to the Andreev reflection of electrons with energy  $eV$  of lead *F*, via the Andreev subgap resonance at energy  $\tilde{\Delta}$  (see Figure 2,b, blue arrow). Importantly, for  $\Delta\varphi = 0$ , the SDIPS-induced

noise steps are absorbed into the noise singularities of the  $\Delta\varphi = 0$  case which are thus reinforced, because  $|D_{\sigma}(\varepsilon)|^2$  is resonant at  $\varepsilon = \pm\Delta$ . When  $\Delta\varphi \neq 0$ , the singularities which existed for  $\Delta\varphi = 0$  are either smoothed, like the singularity at  $\hbar\omega = eV - \Delta$  (marked with ★) and the singularity at  $\hbar\omega = -\Delta$  (marked with +), or even vanish. Measuring the frequencies of the steps appearing in  $S_{ex}$  can give a direct access to the SDIPS parameter  $\Delta\varphi$ , since these frequencies are merely linear combinations of  $\tilde{\Delta}$ ,  $V$ , and  $\Delta$ , independently of any other parameter. Moreover,  $V$  does not affect the positions of all the singularities and steps of  $S_{ex}$  in the same manner (see table in Fig. 1), which can help to identify these features in an experiment. In practice, the frequency range studied in Fig. 1 should be accessible with on-chip detectors, which presently allow to reach  $\omega \sim 250$  GHz<sup>11,20</sup>.

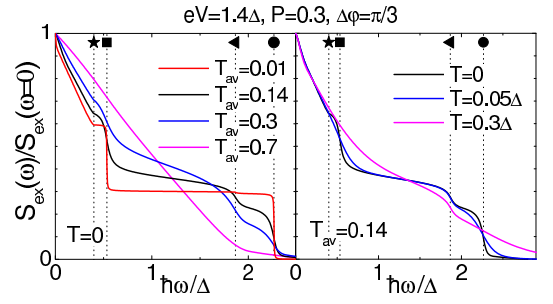


FIG. 3: Excess noise  $S_{ex}$  versus  $\omega$ . The left and right panels show the effect of an increase of the spin-averaged transmission probability  $T_{av}$  and of the temperature  $T$ , respectively. The symbols correspond to frequencies defined in Fig. 1.

The left and right panels of Fig. 3 show how  $S_{ex}$  depends on the spin-averaged transmission probability  $T_{av}$  of the QPC and on the temperature  $T$ , respectively. For brevity, we present results for  $\omega > 0$  only. The singularities and steps displayed by  $S_{ex}$  remain visible for experimentally accessible temperatures (see right panel, blue curve, for  $k_B T = 0.05\Delta$ ). Remarkably,  $T$  and  $T_{av}$  do not influence the visibility of all the steps and singularities of  $S_{ex}$  in the same way. Those occurring for  $\hbar\omega \in \{eV - \Delta, eV \mp \tilde{\Delta}\}$ , marked with ★, ■ and ●, are very sharp when  $T_{av}$  and  $T$  are small (see left panel, red curve). The visibility of these features decrease when  $T_{av}$  or  $T$  increase, due to a smoothing of  $|D_{\sigma}(\varepsilon)|^2$  or of the Fermi factors, respectively. The step occurring at  $\hbar\omega = \Delta + \tilde{\Delta}$ , marked with ◀, has a significantly different behavior. First, it is smoothed by an increase in  $T$ , but much more slowly than the other singularities. This is due to the fact that the dependence of  $S$  on  $\Delta$  and  $\tilde{\Delta}$  occurs through the scattering matrix of the QPC, which does not depend on  $T$ , whereas  $V$  occurs through the Fermi factors. Second, this step vanishes for  $T_{av}$  large but also for  $T_{av}$  small. From a detailed analysis of Eq. (5), this last behavior can be traced to the fact that  $s_{ee}^{\sigma}(\varepsilon)$  is almost constant around  $\varepsilon = \Delta$  at low transparencies.

Finally, we study the dependence of  $S_{ex}$  on the bias

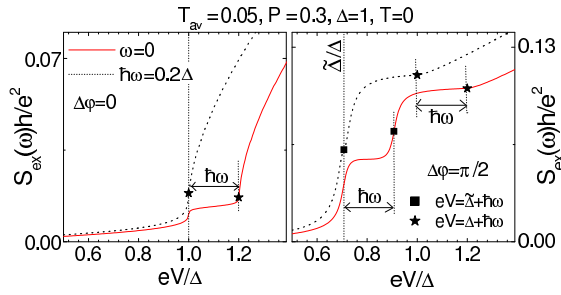


FIG. 4: Excess noise  $S_{ex}$  as a function of the bias voltage  $V$ , for  $\omega = 0$  [ $\omega \neq 0$ ] (dotted [full] lines). We have used no SDIPS on the left panel and a finite SDIPS on the right panel.

voltage  $V$ , which can be measured e.g. using cryogenic low noise amplifiers<sup>6</sup>. The left and right panels of Fig. 4 show curves calculated for a weakly transparent contact, without and with a SDIPS respectively. For simplicity, we focus on the range  $eV < 2\Delta$ . For  $\Delta\varphi = 0$  and  $\omega = 0$ ,  $S_{ex}$  presents a single singularity at  $V = \Delta/e$ . This singularity follows straightforwardly from Eq. (1), but it has not appeared in Fig. 1 because its position is independent of  $\omega$ . For  $\Delta\varphi = 0$  and  $\omega \neq 0$ , a second singularity appears, at  $V = (\Delta + \hbar\omega)/e$ . This feature corresponds to the step at  $\hbar\omega = eV - \Delta$  in Fig. 1. When  $\Delta\varphi \neq 0$ , a single singularity occurs, at  $V = (\Delta + \hbar\omega)/e$ . It is softer than in the case  $\Delta\varphi = 0$ , similarly to what happens for the singularity at  $\hbar\omega = eV - \Delta$  in Fig. 1. Furthermore, two noise step appears at  $V = \tilde{\Delta}/e$  and  $V = (\tilde{\Delta} + \hbar\omega)/e$ . Although cryogenic low noise amplifiers are presently limited to 8 GHz<sup>6</sup>, they should allow to observe the features described in this paragraph since they can occur at frequencies much lower than  $\Delta$ <sup>20</sup>. The SDIPS already modifies qualitatively the noise curve for  $\omega = 0$ , but we think that a measurement at  $\omega \neq 0$  will be crucial in practice. It will allow to confirm the nature

of the singularities and steps occurring in  $S_{ex}$ , and it will also help to calibrate the observed  $\tilde{\Delta}$  and  $\Delta$ , thanks to the scale  $\hbar\omega$  which appears naturally in the noise curves (see Fig. 4, horizontal arrows).

So far, experiments on  $S/F$  point contacts have been performed in the limit of a large number of channels. However, we believe that the few channels regime may be realized, using e.g. a Scanning Tunneling Microscope (STM). Indeed, STMs allow to reach the few channels regime (atomic contact)<sup>21</sup>, and they are particularly suitable for making hybrid contacts. Moreover, current noise has already been measured in a STM<sup>22</sup>. Another possibility could be to contact a carbon nanotube to a  $F$ <sup>23</sup> and a  $S$ <sup>24</sup>. Interestingly, a multichannel ballistic  $S/F$  contact has been considered in Ref. 25, using a rectangular potential barrier model. Although the transverse momentum of particles has been taken into account, the SDIPS is found to be approximately independent of the channel index when  $F$  is strongly polarized<sup>4</sup>. We think that the SDIPS-induced noise steps are bound to persist in these conditions. More generally, in the multichannel case, the occurrence of the SDIPS-induced noise steps will depend on the detailed structure of the contact.

In summary, we have calculated the finite-frequency non-symmetrized current noise of a superconductor/ferromagnet quantum point contact (QPC). For low transparencies, this quantity is qualitatively modified by the Spin-Dependence of Interfacial Phase Shifts, which induces characteristic noise steps at various frequencies and voltages. Finite-frequency noise can thus be an interesting tool to characterize the scattering properties of a  $SF$  QPC.

We acknowledge financial support by the Swiss National Science Foundation, the German Research Foundation (DFG) through SFB 767 and the Landesstiftung Baden-Württemberg. We thank C. Glattli and T. Kontos for discussions.

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  - <sup>16</sup> For  $N/N$  QPCs,  $S_{ex}(\omega) = S_{ex}(-\omega)$  is often found (see e.g. Refs. 9,10), due to the hypothesis of energy independent scattering [see Eq. (7)].
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